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A study of wing-fuselage interaction

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Summary

The problem addressed is that of the initial profile appropriate for the calculation of the boundary layer on the wing at a wing-body junction. The geometry considered is such that the fuselage boundary layer reaches the interaction region still attached and it is shown that when the wing has curvature the boundary-layer component of velocity normal to the wing is turned inviscidly through a right angle to provide a (in general) non-zero profile to initiate the wing calculation.

1. Introduction

The simplest problem in the classical theory of corner flows in a laminar incompressible fluid at high Reynolds number is that of flow along the line of intersection of two flat plates. Carrier [1] first considered this geometry by assuming that a single potential function satisfies the continuity equation. He employed an extension of the Blasius method expecting that far from the corner the solution would be essentially that for a flat plate. In subsequent literature this work has been criticized because the transverse equations of motion were not taken into account. The basic three-dimensionality of such a configuration implies that, due to the interaction of the cross-flows, the flows on the two planes are not independent of each other. Not only are the boundary layers on the planes affected by the cross-flows but simultaneously the leading-order inviscid potential flow is to be corrected. Rubin [2] reconsidered the problem of flow along such a corner emphasising the three-dimensionality and the interaction between four distinct regions of flow in the cross-section of interest. He formulated the problem as a singular perturbation problem obtaining the proper boundary conditions as a result of asymptotic matching similar to that employed by Stewartson [3] in his study of the quarter-infinite plate problem. Explicit results were presented for corrections to the leading-order potential flow and to the boundary-layer solutions away from the corner; discussion of the corner layer was deferred. Pal and Rubin [4] considered the asymptotic behaviour of the corner-layer equations and showed that separate expansions are required to match the corner layer to the potential flow and the corner layer to the boundary layers. They also demonstrated the algebraic decay of the cross-flow velocities both into the boundary layers and the potential flow. Rubin and Grossman [5] dealt with numerical aspects of the

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solution taking proper account of the asymptotic formulae discussed by Pal and Rubin [4] and giving particular attention to the arbitrary constants and logarithmic terms in these expansions. Ghia [6] used a modified transformation of the independent variables such that all transformed quantities in his numerical study remained bounded. The earlier corner-region solutions for the streamwise velocity and the vorticity were reproduced though the secondary-flow streamlines had more curvature than those obtained by Rubin and Grossman [5]. Limit forms for the solutions were used as far-field boundary conditions instead of high-order asymptotic solutions as had been employed by Rubin and Grossman.

In practice, corner flows are greatly influenced by the experimental conditions, the shape of the leading edge and the distribution of streamwise pressure gradient. Also, theoreticians assume the flow to be stable while the experimental evidence of Zamir [7] and El-Camal and Barclay [8] strongly indicate that this is not so. In a recent review, Zamir [9] compares and analyses various theoretical and experimental results and suggests that laminar corner flow does not exist with a zero imposed pressure gradient for Reynolds numbers much above 10^4.

The problem discussed above is essentially that of the three-dimensional adjustment, due to viscosity, of the flow along a corner. On each plane a boundary layer of thickness $O(lR^{-1/2})$ develops from the leading edge in each of which the streamwise velocity is $O(U_\infty)$ and the normal velocity is $O(U_\infty R^{-1/2})$. Here R is a Reynolds number based on a typical length l and the streamwise external velocity U_∞ at upstream infinity. In the corner region of $O(lR^{-1/2})$ by $O(lR^{-1/2})$ in extent the secondary velocities are again $O(U_\infty R^{-1/2})$. A rather different type of corner problem has recently been discussed by Smith and Duck [10]. They envisage the collision of opposing jets or the turning of a thermal boundary layer on encountering a concave corner. The theory is two-dimensional and interactive and the authors propose that the oncoming jet or boundary layer of thickness $O(lR^{-1/2})$ anticipates the presence of the corner and separates at a distance $O(lR^{-3/14})$ upstream of it in an interaction region of length $O(lR^{-3/7})$. The boundary layer leaves the wall as a free shear layer underneath which there is an eddy of recirculating fluid. However, suggestions that this theory is applicable to the colliding boundary layers near the equator of a spinning sphere are not supported by the numerical investigations of Dennis, Singh and Ingham [11] who find no sign of separation at Reynolds numbers up to 100. There is no sign of separation in the experiments either (Bowden and Lord [12]).

Our aim in this work is to shed some light on the problem of wing-fuselage interaction in laminar incompressible flow at large Reynolds number. In particular, we wish to examine the flow in the neighbourhood of the root chord as this represents an initial line for the three-dimensional boundary layer that develops along the length of the wing. It is expected that this flow depends crucially on the design of the interaction region. If the fuselage extends well ahead of the wing the most likely configuration is that of the complex horse-shoe vortex resulting from the separation of the fuselage boundary layer before the wing is encountered. This may be laminar in appearance (El-Gamal and Barclay [8]) or clearly turbulent as in the experiments of East and Hoxey [13]. A less dramatic possibility for separation of the fuselage boundary layer, but which also cannot be ruled out, is that of the turning process of Smith and Duck [10] discussed above. A third possibility, and the one that is proposed here, occurs when the design of the interaction region is such that the fuselage boundary layer reaches the wing still attached. The geometry chosen here has the fuselage given by the half plane $z = 0$, $x \geqslant 0$ and the wing by the cylinder $y = \pm f(x)$, with f(0) = 0, so that the fuselage does not extend ahead of the wing for an oncoming flow in the x-direction. This geometry itself does not necessarily preclude separation but for the example we chose it did not occur.

The velocity profiles of the fuselage boundary layer when it reaches the wing are of fundamental importance for a successful solution for the flow over the complete wing-body structure, for they provide, in part, the initial conditions for the next stage of the computation, namely the boundary layer on the wing. It is often assumed that, in the limit of large Reynolds number, the initial profile for integration along the length of wing is zero. We shall see that this assumption is correct when the wing consists of a wedge but is incorrect if the cross-section of the wing has curvature. In the former case the wedge, which is of course a streamsurface of the inviscid flow, is also a streamsurface of the boundary layer on the fuselage. The boundary layer meets the wedge with a zero velocity component normal to the wedge and thus the secondary velocities in the corner region are $O(U_\infty R^{-1/2})$. This means that in this situation the corner problem is analogous to that of the intersecting flat plates considered by Carrier [1] by Rubin and Grossman [5] and by Zamir [9] as discussed above. However, when the wing section is circular, or indeed has any curvature, it follows, as noted by Rosenhead [14], that the streamlines of the boundary layer cannot be parallel to those of the inviscid flow (of which the wing section is one). Thus, unless it separates ahead of the wing, the boundary layer must reach the wing with a velocity component normal to it that is nonzero. We show for the particular case of a circular cylinder that this is indeed what happens and, in addition, that this nonzero profile is turned through a right angle in an inviscid region that is $O(lR^{-1/2})$ by $O(lR^{-1/2})$ to provide an initial profile for the incipient wing boundary layer. The boundary layer for this wing will be fully three-dimensional, as the wing also presents a leading edge to the oncoming flow, and its solution has not been attempted here.

Calculations for the fully three-dimensional situation in which a vertical cylinder is mounted on a horizontal flat plate at a non-zero distance from the leading edge were carried out by Sowerby [25] by the method of series expansions. Although the limited number of terms available did not make it possible either to predict a horseshoe vortex or to extend the calculation as far as the boundary of the cylinder, non-negligible deviation of the boundary-layer streamlines from those of the inviscid flow was clearly demonstrated.

The phenomenon that we have described above may be termed a collision in that the fuselage boundary layer meets the wing with a nonzero normal velocity. It is, however, rather different from other collisions, both steady and unsteady, that have recently been the subject of many investigations, in that the calculation does not terminate with a singularity. A viscous singularity was encountered by Stewartson and Simpson [15] (see also Stewartson, Cebeci and Chang [16]) in their examination of the steady entry flow on the line of symmetry on the inside of a curved pipe. The computation comes to an end at the position of zero axial skin friction at which point the axial boundary layer leaves the wall and the azimuthal boundary layers collide underneath it. A similar singularity occurs on the leeside of a cone at incidence (Cebeci, Stewartson and Brown [17]) and at the upper pole of a heated sphere in a fluid at rest (Brown and Simpson [18]). Each of these three examples differ from that considered here in that they are fluid-fluid collisions, not fluid-boundary collisions. Unsteady collision phenomena in which the singularity occurs at a finite time in the outer inviscid part of the boundary layer have also recently been examined. Examples of these are the collision of opposing boundary layers at the equator of an impulsively started spinning sphere (Simpson and Stewartson [19]), at the centre of an impulsively started rotating disk in a counter-rotating fluid (Banks and Zaturska [20], Stewartson, Simpson and Bodonyi [21]), at the highest generator of an impulsively heated cylinder (Simpson and Stewartson [22]), and at the upper pole of an impulsively heated sphere (Brown and Simpson [18]). The present collision provides information for the continuation of the calculation and does not signal the approach of a time or position at which the boundary-layer equations have become inadequate.

2. The geometry and the equations of motion

Let us fix the class of problems we shall consider here by defining the rigid surfaces to be $z=0$, $x \geqslant 0$ and $y = \pm f(x)$, $x \geqslant 0$, $z > 0$, where $f(x)$ is a given function of x which vanishes at $x=0$. In Fig. 1 we give a sketch of the configuration in the x, y-plane. The plane $z=0$ represents the fuselage and the cylinder the wing. The oncoming flow is inviscid for $x<0$ and at $x=0$ two boundary layers are set up, one on the cylinder $y = \pm f(x)$, $z>0$ and one on the plane $z=0$ but outside the cylinder. Intuitively the first of these appears to be a straightforward two-dimensional boundary layer of the classical kind but we shall see that this concept must be modified due to a mass flux from the second. We focus our interest in this chapter on the second boundary layer, namely the boundary layer which is set up on the plane $z=0$.

A discussion of three-dimensional boundary-layer effects may be found in Rosenhead [14]. There it is shown that if the inviscid external streamlines have nonzero lateral curvature the streamlines in the boundary layer cannot be parallel to them. This is the situation that prevails here; if u_e, v_e, w_e are the components of the external velocity then on the cylinder these take the values

$$w_e = 0, \quad v_e = \pm f'(x) u_e, \quad \text{when } y = \pm f(x). \tag{2.1}$$

Thus the cross-section of the cylinder is a streamline of the external flow and, except in the particular case when it is a wedge and has zero curvature, it cannot also be a streamline of the boundary layer on the plane $z=0$. This follows either from the

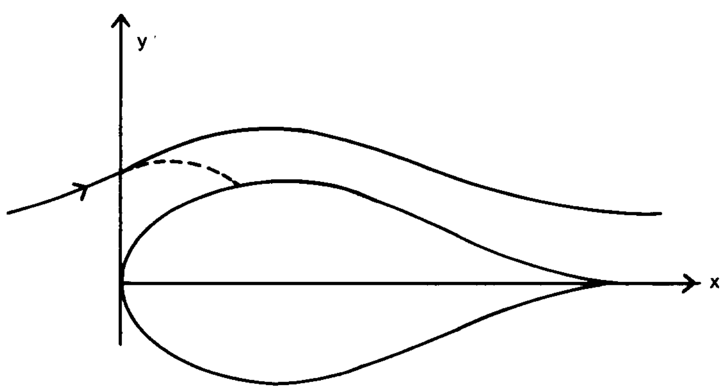

Figure 1. The geometry of the interaction region. ——— external streamline - - - - - - boundary-layer streamline.

argument of Rosenhead or, equivalently and directly, by noting that if the cylinder were also a boundary-layer streamline we should have

$$\frac{\partial p}{\partial n} = \rho \kappa u_s^2, \tag{2.2}$$

at points in the boundary layer just outside the cylinder. Here p is the pressure, ρ the density, n the outward drawn normal to the cylinder, κ the curvature of the cylinder and u_s the component of velocity parallel to the cylinder. Now the left-hand side of (2.2), and κ, are independent of z, as p is given by the main stream as $z \to 0$, whereas u_s itself is a non-constant function of z since it vanishes at $z = 0$. It follows therefore that the relation (2.2) cannot be satisfied unless $\kappa = 0$ in which case the cylinder reduces to a wedge. The most likely resolution of this contradiction is either that the oncoming boundary layer on the plane $z = 0$ separates before the cylinder is reached or that the component of velocity normal to the cylinder is nonzero at the cylinder. This second situation, which we shall demonstrate is the relevant one, we term a collision phenomenon.

The first example we shall choose has $f(x) = \pm x \tan \alpha$ where $0 < \alpha < \pi/2$ and so $\kappa = 0$, and the argument based on (2.2) does not apply. Hence we might expect that the question of a collision does not arise and the purpose of this study is to show that separation does not occur either. In the second example $f(x) = \sqrt{x(l - x)}$ where l is a constant and x is assumed to be small. Now the curvature is κ $(= 2/l)$ and we show that here too there is no separation but that collision clearly occurs at the cylinder. The boundary-layer equations for both these problems are the same, namely

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= u_e\frac{\partial u_e}{\partial x} + v_e\frac{\partial u_e}{\partial y} + \nu\frac{\partial^2 u}{\partial z^2}, \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= u_e\frac{\partial v_e}{\partial x} + v_e\frac{\partial v_e}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \end{aligned} \tag{2.3}$$

where ν is the kinematic viscosity, and the boundary conditions are that

$$\begin{aligned} &u = v = w = 0 \ \text{ at } z = 0,\ x > 0,\ |y| > f(x), \\ &u \to u_e, \quad v \to v_e \ \text{ as } z \to \infty, \end{aligned} \tag{2.4}$$

and the boundary layer has zero thickness at $x = 0$ as the onset of the flow in both cases is of Blasius type.

3. The wedge

In practice, the wedge-shape $y = \pm x \tan \alpha$, $x \geqslant 0$, is relevant to the front portion of a finite sharp-nosed cylinder. The structure of the flow in this neighbourhood is determined solely by the angle α through which the flow has to turn. It is convenient to introduce cylindrical coordinates (r, θ, z) where $r \cos \theta = x$, $r \sin \theta = y$. With velocity components

(u_r, u_θ, u_z) in this system of coordinates the boundary-layer equations (2.3) take the form

$$u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}=\nu\frac{\partial^2 u_r}{\partial z^2}+u_{re}\frac{\partial u_{re}}{\partial r}+\frac{u_{\theta e}}{r}\frac{\partial u_{re}}{\partial \theta}+\frac{1}{r}\left(u_\theta^2-u_{\theta e}^2\right),$$

$$u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}=\nu\frac{\partial^2 u_\theta}{\partial z^2}+u_{re}\frac{\partial u_{\theta e}}{\partial r}+\frac{u_{\theta e}}{r}\frac{\partial u_{\theta e}}{\partial \theta}-\frac{1}{r}\left(u_r u_\theta-u_{re}u_{\theta e}\right),$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0, \tag{3.1}$$

where the corresponding components $(u_{re}, u_{\theta e})$ of the inviscid flow may be written as

$$u_{re}=-r^n\cos q,\quad u_{\theta e}=-r^n\sin q, \tag{3.2}$$

with $(n+1)(\pi-\alpha)=\pi$, and $q=(n+1)(\pi-\theta)$, and the boundary conditions are

$$u_r=u_\theta=u_z=0\quad \text{at } z=0,\quad u_r\to u_{re},\quad u_\theta\to u_{\theta e}\quad \text{as } z\to\infty. \tag{3.3}$$

In the boundary layer we write

$$\tilde{\xi}=\left(r^{n-1}/\nu\cos\theta\right)^{1/2}z,\quad n-1<0,$$

$$u_r=-r^n\tilde{U}(\tilde{\xi},\theta)\cos q,\quad u_\theta=-r^n\tilde{V}(\tilde{\xi},\theta)\sin q,$$

$$u_z=\left(\nu r^{n-1}/\cos\theta\right)^{1/2}$$

$$\times\left[\tilde{W}(\tilde{\xi},\theta)+(1/2)(n-1)\tilde{\xi}\tilde{U}\cos\theta\cos q+(1/2)\tilde{\xi}\tilde{V}\sin\theta\sin q\right], \tag{3.4}$$

whereupon the governing equations (3.1) reduce to

$$\frac{\partial^2\tilde{U}}{\partial\tilde{\xi}^2}-\tilde{W}\frac{\partial\tilde{U}}{\partial\tilde{\xi}}$$

$$=-\frac{\cos\theta}{\cos q}\left[\tilde{V}\frac{\partial\tilde{U}}{\partial\theta}\sin q\cos q+(n+1)\tilde{U}\tilde{V}\sin^2 q+n\tilde{U}^2\cos^2 q-\tilde{V}^2\sin^2 q-n\right],$$

$$\frac{\partial^2\tilde{V}}{\partial\tilde{\xi}^2}-\tilde{W}\frac{\partial\tilde{V}}{\partial\tilde{\xi}}=-\cos\theta\left[\tilde{V}\frac{\partial\tilde{V}}{\partial\theta}\sin q-(n+1)(\tilde{V}^2-\tilde{U}\tilde{V})\cos q\right],$$

$$\frac{\partial\tilde{W}}{\partial\tilde{\xi}}+\tfrac{1}{2}\tilde{V}\sin\theta\sin q=-\cos\theta\cos q\left[(n+1)\tilde{V}-\tfrac{1}{2}(n+3)\tilde{U}-\frac{\partial\tilde{V}}{\partial\theta}\tan q\right], \tag{3.5}$$

and the boundary conditions (3.3) become

$$\tilde{U}=\tilde{V}=\tilde{W}=0\ \text{at } \tilde{\xi}=0,\quad \tilde{U}\to 1,\quad \tilde{V}\to 1\ \text{as } \tilde{\xi}\to\infty. \tag{3.6}$$

Table 1. Reduced skin friction for three-dimensional boundary-layer flow upstream of a wedge

$\alpha = 30°$			$\alpha = 45°$		$\alpha = 60°$	
$\theta°$	$\tilde{S}_B$	$\tilde{T}_B$	$\tilde{S}_B$	$\tilde{T}_B$	$\tilde{S}_B$	$\tilde{T}_B$
90	0.324	0.324	0.309	0.309	0.279	0.279
80	0.387	0.321	0.395	0.302	0.398	0.262
70	0.420	0.315	0.456	0.290	0.510	0.231
60	0.446	0.308	0.518	0.272	0.675	0.197
50	0.478	0.298	0.615	0.248		
45	0.501	0.291	0.685	0.249		
40	0.532	0.284				
30	0.634	0.28				

Initially, at $\theta = \pi/2$ where $q = \pi^2/2(\pi - \alpha)$, the flow field is essentially of Blasius type, and the solution is extended numerically to smaller values of θ, i.e. larger values of q, by the standard Keller-box method, fully documented in Cebeci and Bradshaw [23]. The step lengths chosen were 0.15 in ξ and 5° in θ; the outer edge of the boundary was taken at $\xi = 9$. No problems were experienced in the computation, the value of $\tilde{V}$ being clearly positive for all $\xi > 0$ in the range of θ of interest, and, as $\theta \to \alpha$, it remained finite. Hence we see from (3.4) that the component of velocity in the boundary layer, u_θ, normal to the wedge, tends to zero uniformly as $\theta \to \alpha$ and $(n+1)(\pi - \theta) \to \pi$. There is no separation ahead of the wedge and no collision of the plate boundary layer with the wedge, and the way is open therefore to study the interaction between the two boundary layers, one on the cylinder and one on the plane $z = 0$, in a similar manner to that for the corner region of Rubin as discussed in the introduction.

To illustrate the results of the computation of the plate boundary layer undertaken here we display in Table 1 the components of the reduced skin friction

$$\tilde{S}_B = \left.\frac{\partial \tilde{U}}{\partial \xi}\right|_{\xi=0}, \quad \tilde{T}_B = \left.\frac{\partial \tilde{V}}{\partial \xi}\right|_{\xi=0}, \tag{3.7}$$

as functions of θ for $\alpha = 30°, 45°, 60°$.

The only other feature of interest is that very near $\theta = \alpha$ a slight overshoot, of a fraction of a percentage, develops in the profile of $\tilde{U}$. This is possibly related to the slight increase of the θ component of skin friction as the wedge is attained also evident from Table 1.

4. The circular cylinder

If the rigid surface standing on the plane $y = 0$ is the circular cylinder $y^2 = x(l - x)$, with centre at $(l/2, 0, 0)$ and radius $l/2$, the inviscid velocity distribution above the plane $z = 0$ is given by

$$u_e = \partial\phi/\partial x, \quad v_e = \partial\phi/\partial y, \quad w_e = 0 \text{ where}$$

$$\phi = U_\infty\left(x - \tfrac{1}{2}l\right)\left[1 + \frac{l^2/4}{\left(x - \frac{1}{2}l\right)^2 + y^2}\right]. \tag{4.1}$$

Near the forward stagnation point $x=y=0$ and in the region $x \geqslant 0,\ y^2 \geqslant x(l-x)$,

$$u_e = 4U_\infty(3y^2 - lx)/l^2 + O(y^4), \quad v_e = 4U_\infty y/l + O(y^3); \tag{4.2}$$

we must bear in mind that $y^2 = O(lx)$ in this region. A solution of the boundary-layer equations (2.3) may be found by writing

$$\begin{gathered} \xi = \frac{2z}{\sqrt{\zeta}}\sqrt{\frac{U_\infty}{\nu l}}, \quad \zeta = \frac{lx}{y^2}, \quad Y = \frac{y}{l}, \\ u = 4Y^2 U_\infty U(\xi, \zeta, Y), \quad v = 4YU_\infty V(\xi, \zeta, Y), \\ w = \frac{2}{\sqrt{\zeta}}\sqrt{\frac{U_\infty \nu}{l}}\,[W(\xi, \zeta, Y) + (1/2)\xi U - \xi\zeta V]. \end{gathered} \tag{4.3}$$

The equations satisfied by U, V, W are free of singularities near $Y=0$ and are given by

$$\begin{gathered} \frac{\partial W}{\partial \xi} + \tfrac{1}{2}U = 2\zeta^2\frac{\partial V}{\partial \zeta} - \zeta\frac{\partial U}{\partial \zeta} - \zeta Y\frac{\partial V}{\partial Y}, \\ \frac{\partial^2 U}{\partial \xi^2} - W\frac{\partial U}{\partial \xi} = \zeta\left[\frac{\partial U}{\partial \zeta}(U - 2\zeta V) + 2UV + YV\frac{\partial U}{\partial Y}\right] - \zeta^2 - 3\zeta, \\ \frac{\partial^2 V}{\partial \xi^2} - W\frac{\partial V}{\partial \xi} = \zeta\left[\frac{\partial V}{\partial \zeta}(U - 2\zeta V) + V^2 + YV\frac{\partial V}{\partial Y}\right] - \zeta. \end{gathered} \tag{4.4}$$

A formal solution to these equations may be found by expanding U, V, W in integer powers of Y^2. The leading terms (U_0, V_0, W_0) satisfy

$$\begin{gathered} \frac{\partial W_0}{\partial \xi} + \tfrac{1}{2}U_0 = 2\zeta^2\frac{\partial V_0}{\partial \zeta} - \zeta\frac{\partial U_0}{\partial \zeta}, \\ \frac{\partial^2 U_0}{\partial \xi^2} - W_0\frac{\partial U_0}{\partial \xi} = \zeta\left[\frac{\partial U_0}{\partial \zeta}(U_0 - 2\zeta V_0) + 2U_0V_0\right] - \zeta^2 - 3\zeta, \\ \frac{\partial^2 V_0}{\partial \xi^2} - W_0\frac{\partial V_0}{\partial \xi} = \zeta\left[\frac{\partial V_0}{\partial \zeta}(U_0 - 2\zeta V_0) + V_0^2\right] - \zeta, \end{gathered} \tag{4.5}$$

with boundary conditions

$$U_0 = V_0 = W_0 \text{ at } \xi = 0 \quad \text{and } U_0 \to 3 - \zeta,\ V_0 \to 1 \text{ as } \xi \to \infty. \tag{4.6}$$

The solution begins at $\zeta = 0$, i.e. the leading edge $x=0$ of the plate, where the flow field is again of Blasius form, and continues until $\zeta = 1$ which coincides with the boundary of the circular cylinder when $Y \ll 1$. More generally this boundary is $\zeta = 1 + \zeta^2 Y^2$. The numerical integration is again performed using the Keller-box method with step sizes of 0.08 in ξ, and varying from 0.1 in ζ for $0 \leqslant \zeta \leqslant 0.8$ down to 0.01 near $\zeta = 1$ with the outer boundary taken at $\xi = 4.88$. No difficulties were encountered in continuing the integration as far as $\zeta = 1$; in particular there was no sign of separation. In Table 2 we display the components of the reduced skin friction for the case of the circular cylinder and there is clearly no indication of their being zero or singular anywhere in $0 \leqslant \zeta \leqslant 1$.

Table 2. Reduced skin friction for three-dimensional boundary-layer flow upstream of a circular cylinder

ζ	S_B	T_B
0.0	1.727	0.576
0.1	1.879	0.645
0.2	2.038	0.713
0.3	2.198	0.779
0.4	2.366	0.845
0.5	2.537	0.910
0.6	2.718	0.975
0.7	2.903	1.039
0.8	3.099	1.103
0.9	3.303	1.168
1.0	3.517	1.233

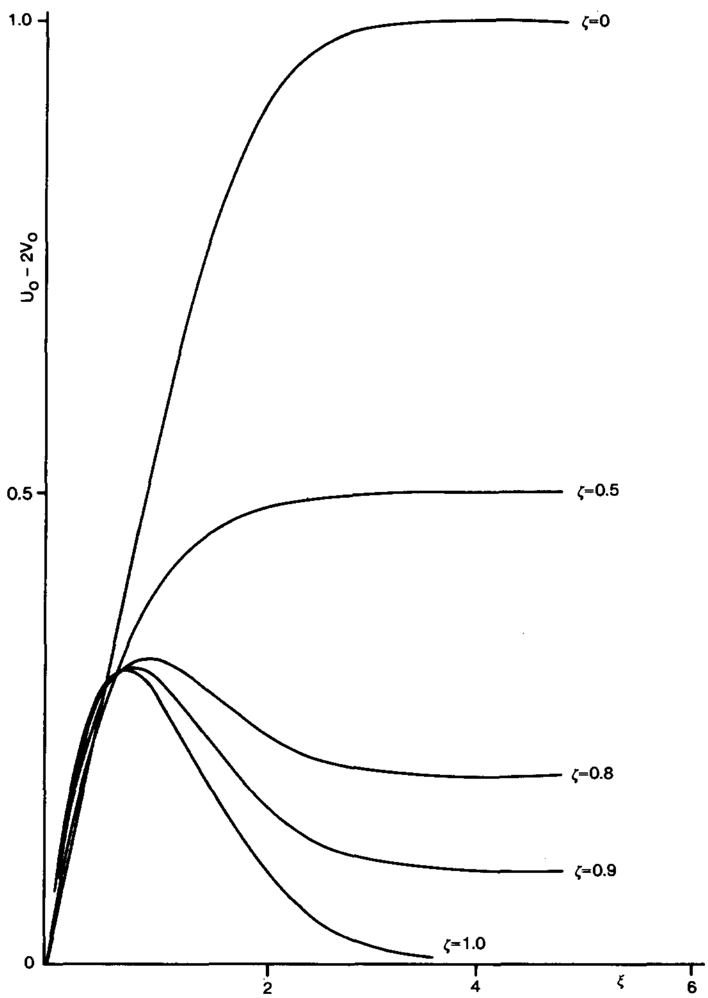

Figure 2. Profiles of $U_0 - 2V_0$ for various values of ζ.

The properties of $U_0 - 2V_0$ are of principal interest since the component of velocity normal to the cylinder is given by $4Y^2U_\infty[2V - U + 2\zeta Y^2 U]$, and in Fig. 2 we plot profiles of $U_0 - 2V_0$ for various values of ζ. It is clear that, as expected, this velocity component is not zero for all $\xi > 0$ at the circular cylinder though it duly vanishes at the plane $z = 0$ and in the external stream. Hence, according to our theory, the fluid in the boundary layer on the plane $z = 0$ moves towards the circular cylinder and collides with it. A sketch of a typical streamline in this boundary layer is shown in Fig. 1. In Table 2 we display the components of the reduced skin friction

$$S_B = \left.\frac{\partial U}{\partial \xi}\right|_{\xi=0}, \quad T_B = \left.\frac{\partial V}{\partial \xi}\right|_{\xi=0}, \tag{4.7}$$

for a circular cylinder. As in the case of a wedge, as $\zeta \to 1$ overshoot of the U-component of velocity occurs here also, rather more pronounced this time.

5. Discussion of the collision process

The numerical studies of these two flows indicate very strongly that for general shapes $f(x)$ with $f''(x) \neq 0$ the boundary layer on the plane $z = 0$ will bring fluid to the cylinder with a nonzero normal velocity so that it must in a sense collide with the cylinder. Separation beforehand does not appear to be an option and while collision does not occur on a wedge our arguments show that this is because its curvature is zero. As soon as some curvature develops the collision is inevitable. For example if the cylinder is a double arc,

$$y = \pm x \tan \alpha\{1 - (x/l)\}, \tag{5.1}$$

then the solution in Section 3 is the leading term in an expansion in ascending powers of x and there is little doubt that the higher terms in the series will contribute to a collision. It is possible that separation will also occur eventually, perhaps near the rear of the cylinder, but this remains to be established.

Even when studied in the most naive way possible, the mechanics of the collision process are quite different from those of the corner flows reviewed by Zamir [9] and discussed here in the introduction. In Zamir's problem the cross-flow velocities are $O(U_\infty R^{-1/2})$, while the streamwise component of velocity is $O(U_\infty)$. In addition, the core region extends a distance $O(lR^{-1/2})$ from the corner so that $\partial/\partial y$, $\partial/\partial z \gg \partial/\partial x$. As a result there is a subtle interplay between all three components of velocity, viscous forces cannot be neglected, and a vital part is played by the streamwise vorticity.

In the present instance the cross-flow is $O(U_\infty)$ and if we assume that the collision region extends no more than a distance $O(lR^{-1/2})$ from the corner, the component of velocity along the corner does not play a significant rôle in controlling the turning of the fluid from a motion parallel to the z-plane to a motion parallel to the cylinder. Let us define a new system of coordinates (s, n, ξ), where s denotes distance along the cylinder parallel to the z-plane, n denotes distance along the outward drawn normal to the cylinder and ξ, as before, measures distance normal to the plane $z = 0$. Let $(\bar{U}, \bar{V}, \bar{W})$ be the corresponding components of velocity in the collision zone and set $\eta = nR^{1/2}/l$. Then boundary-layer theory of the kind we have considered hitherto in this paper would tell us that if ξ is finite

$$\bar{U} \to U_\infty \hat{U}(\xi, s), \quad \bar{V} \to -U_\infty \hat{V}(\xi, s), \quad \bar{W} \approx U_\infty R^{-1/2} \tag{5.2}$$

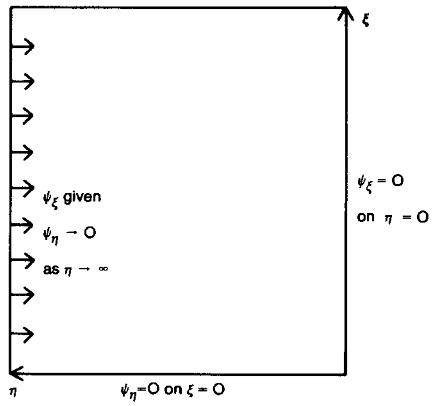

Figure 3. Illustration of the boundary conditions for equations (5.4).

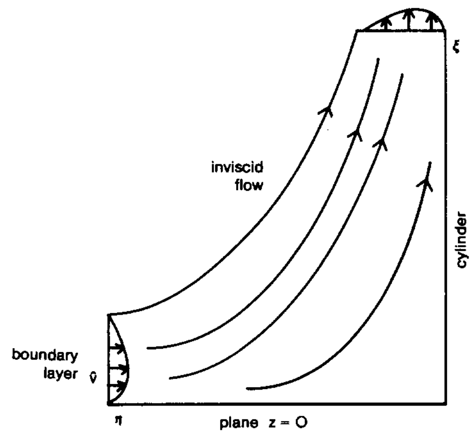

Figure 4. Cross-sectional flow fields at the corner.

as $\eta \to \infty$, where $\hat{U}$, $\hat{V}$, both positive, are calculated from the values of U and V evaluated at the boundary of the circular cylinder $\zeta = 1$ ($Y \ll 1$) and thus fixed by this theory. For discussion of the collision zone, illustrated here for the case of a circular cylinder, we substitute $r = a + l\eta R^{-1/2}$, $z = l\xi R^{-1/2}$, $\theta = s/a$, where $a = l/2$ is the radius of the cylinder, and $u_r = \overline{V}$, $u_\theta = \overline{U}$, $u_\zeta = \overline{W}$ into the full Navier-Stokes equations in polar form and let $\nu \to 0$ ($\equiv R \to \infty$) obtaining the inviscid equations

$$\begin{aligned}&\frac{\partial \overline{V}}{\partial \eta} + \frac{\partial \overline{W}}{\partial \xi} = 0, \quad \overline{V}\frac{\partial \overline{V}}{\partial \eta} + \overline{W}\frac{\partial \overline{V}}{\partial \xi} + \frac{1}{\rho}\frac{\partial \overline{P}}{\partial \eta} = 0,\\ &\overline{V}\frac{\partial \overline{W}}{\partial \eta} + \overline{W}\frac{\partial \overline{W}}{\partial \xi} + \frac{1}{\rho}\frac{\partial \overline{P}}{\partial \xi} = 0, \quad \overline{V}\frac{\partial \overline{U}}{\partial \eta} + \overline{W}\frac{\partial \overline{U}}{\partial \xi} = 0.\end{aligned} \tag{5.3}$$

Equations (5.3) also apply for the corner region of any cylindrical cross-section provided that s measures distance around the circumference and η that along the normal with corresponding velocity components $\overline{U}$, $\overline{V}$. Thus the secondary motion can be computed independently of the primary motion and in effect their rôles are reversed. Further, the secondary motion is controlled by inertia forces only and has already been studied by Stewartson [24] in a related problem. A sketch of the cross-sectional flow-field is shown in Fig. 4.

Equations (5.3) have the formal solution

$$\overline{V} = \partial\psi/\partial\xi, \quad \overline{W} = -\partial\psi/\partial\eta, \quad \frac{\partial^2\psi}{\partial\eta^2} + \frac{\partial^2\psi}{\partial\xi^2} = H(\psi, s), \quad \overline{U} = G(\psi, s), \tag{5.4}$$

with boundary conditions on ψ

$$\begin{aligned}&\frac{\partial\psi}{\partial\xi} \to -U_\infty\hat{V}, \quad \frac{\partial\psi}{\partial\eta} \to 0 \text{ as } \eta \to \infty,\\ &\frac{\partial\psi}{\partial\xi} = 0 \text{ at } \eta = 0, \quad \frac{\partial\psi}{\partial\eta} = 0 \text{ at } \xi = 0.\end{aligned} \tag{5.5}$$

Here H, G are functions of ψ, and s appears only as a parameter. These functions can be fixed by the conditions (5.2) as ψ becomes independent of η as $\eta \to \infty$ since (5.2) requires $\overline{W}$ to be small. Conditions (5.5) are sufficient to determine ψ in (5.4). Thus, just as ψ is independent of η as $\eta \to \infty$ so it is independent of ξ as $\xi \to \infty$ because of the symmetry in the boundary conditions, as shown in Fig. 3. We may refer, therefore, that the rôles of ξ and η are reversed and the rôles of $\overline{V}$ and $-\overline{W}$ are interchanged. Indeed, it is clear from Fig. 3 that the flow field as $\xi \to \infty$ is the same as it is as $\eta \to \infty$, but turned through a right angle. Hence, as $\xi \to \infty$,

$$\overline{U} \to U_\infty \hat{U}(\eta, s), \quad \overline{V} \to 0, \quad \overline{W} \to U_\infty \hat{V}(\eta, s). \tag{5.6}$$

We thus come to the conclusion that the details of the mechanics of turning the corner are of local interest only and the boundary layer which collided with the cylinder has the same velocity distribution as the boundary layer which begins to evolve on the surface of the cylinder, apart from the interchange of ξ and η and of $\overline{V}$ and $-\overline{W}$.

6. Discussion

The present investigation has shown that the common practice of initiating the boundary-layer flow on a wing by simply assuming a zero velocity profile at root chord is not in general correct. In particular, if the fuselage does not extend ahead of the wing the fuselage boundary layer plays an important rôle in determining the velocity profiles on the wing. Here we considered a fuselage that is plane with a cylindrical wing normal to it, and have computed the three-dimensional boundary layer on the plane. Unless the wing cross-section has zero curvature the component of velocity in this boundary layer has a velocity component normal to the wing that does not vanish as the wing is attained. Thus a turning process is inevitable at the intersection and a simple theory suggests that the important effect is to turn the fluid velocity through a right angle in an inviscid manner without any change in profile. The resulting velocity profile may then be used as an initial condition to compute the boundary layer on the cylinder.

If the fuselage extends ahead of the wing it is expected that the fuselage boundary layer will separate upstream of the wing. In such a situation the collision process described here is of secondary importance and the well-known horse-shoe vortex will probably result, though the separation mechanism of Smith and Duck [10] cannot altogether be ruled out. If, however, the region of separation is designed so as to eliminate the horse-shoe vortex then the phenomenon of collision as described here may be significant.

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The problem of colliding boundary layers was one of the topics on which Keith Stewartson was working when he died in May 1983. The present study was initiated by him and completed after his death and both authors acknowledge their indebtedness to him.

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